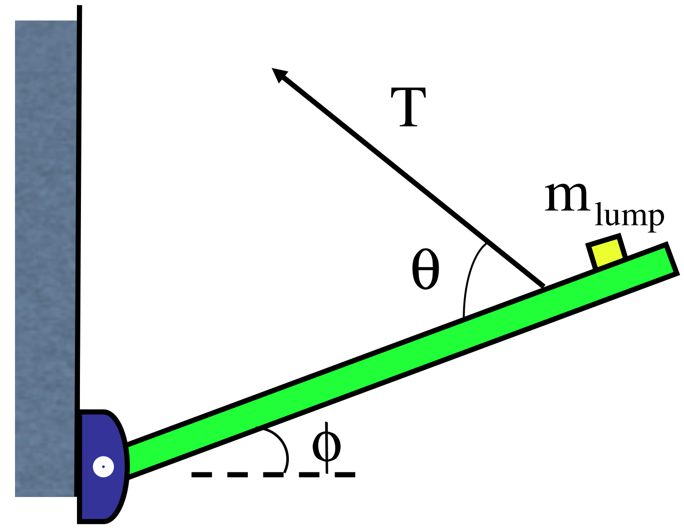
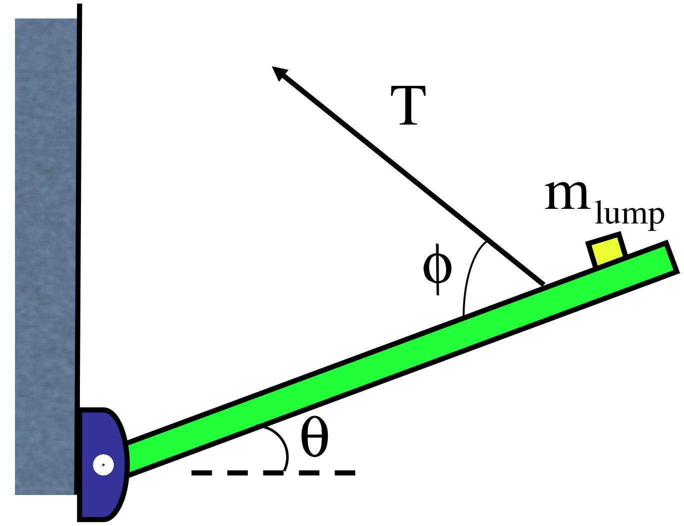
NOTE THAT WITH THE EXCEPTION OF THE PIN PLACEMENT AND THE EXTRA LUMP, BOTH OF WHICH YOU SHOULD BE ABLE TO HANDLE, THIS PROBLEM IS FOR LATER PRACTICE AS IT'S SOLUTION IS VERY SIMILAR TO THAT OF THE MORE INTERESTING PROBLEM 8!

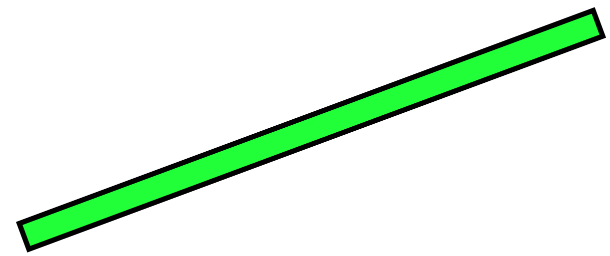


2.) A beam of length "L" is pinned at an angle θ to a wall (see sketch). Tension in a rope "3L/4" from the pin keeps it in equilibrium. There is a massive lump a distance "5L/6" units up the beam. What is known is:



$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

a.) Draw a f.b.d. for the forces on the beam.

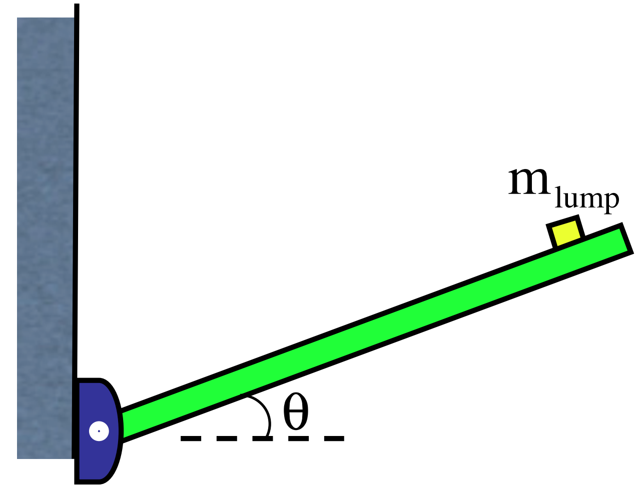


b.) What must the tension in the rope be for equilibrium?

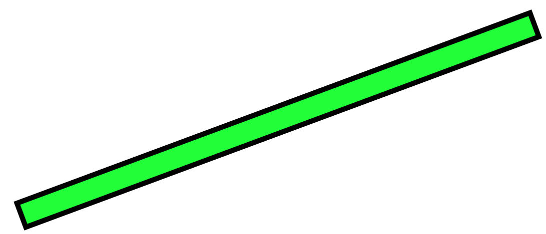
c.) Use the Parallel Axis Theorem to determine the *moment of inertia* of the beam about the pin.

$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12}m_b L^2$$

d.) Determine the *moment of inertia* of the lump, then system, about the pin.

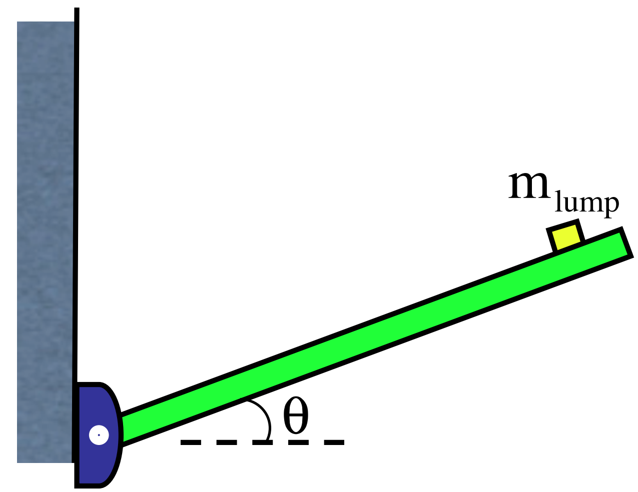


e.) The rope is cut and the beam begins to angular accelerate downward. What is the beam's initial angular acceleration?

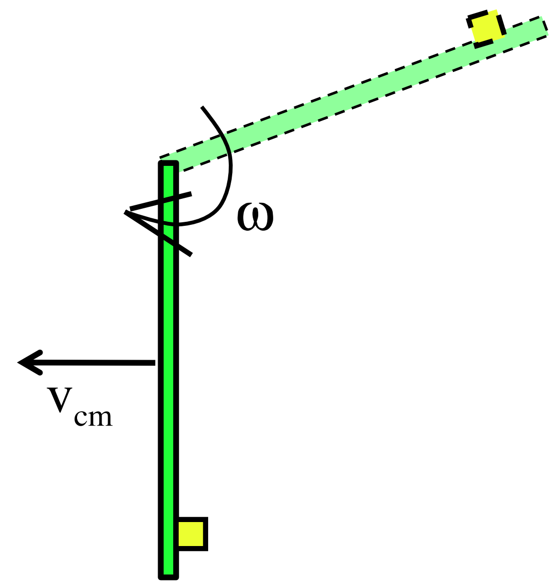


$$m_{beam}\,,\ m_{lump}\,,\ L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

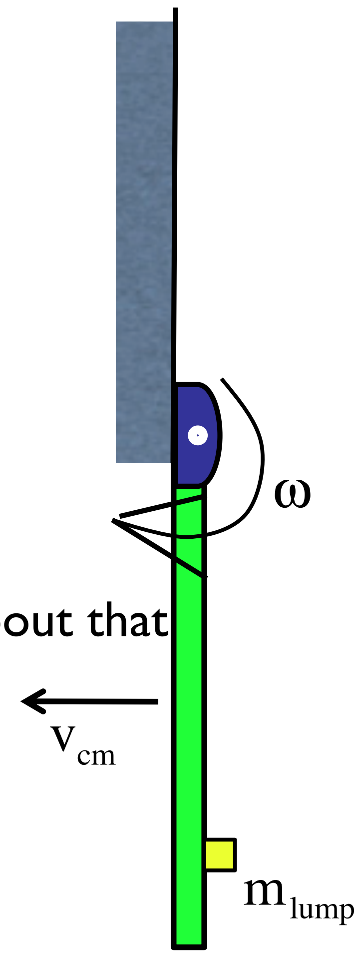
f.) What is the lump's initial acceleration?



g.) The beam rotates about the pin. What is its angular velocity as it passes through the vertical?



$$m_{beam}, m_{lump}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

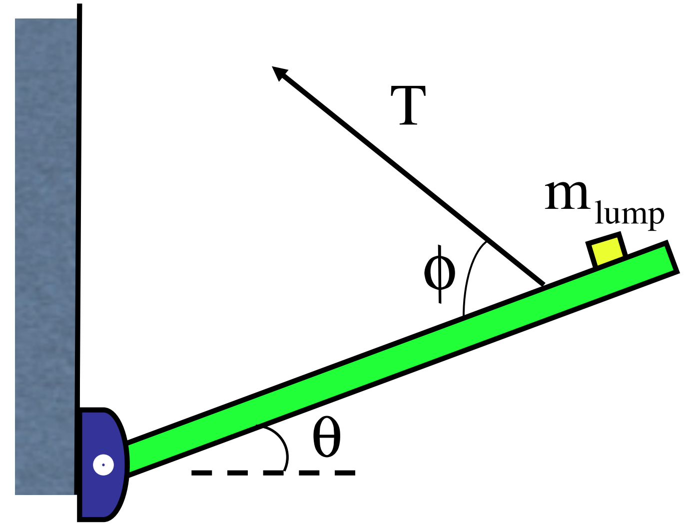


h.) What is the beam's *center-of-mass* velocity as it passes through the bottom of the arc?

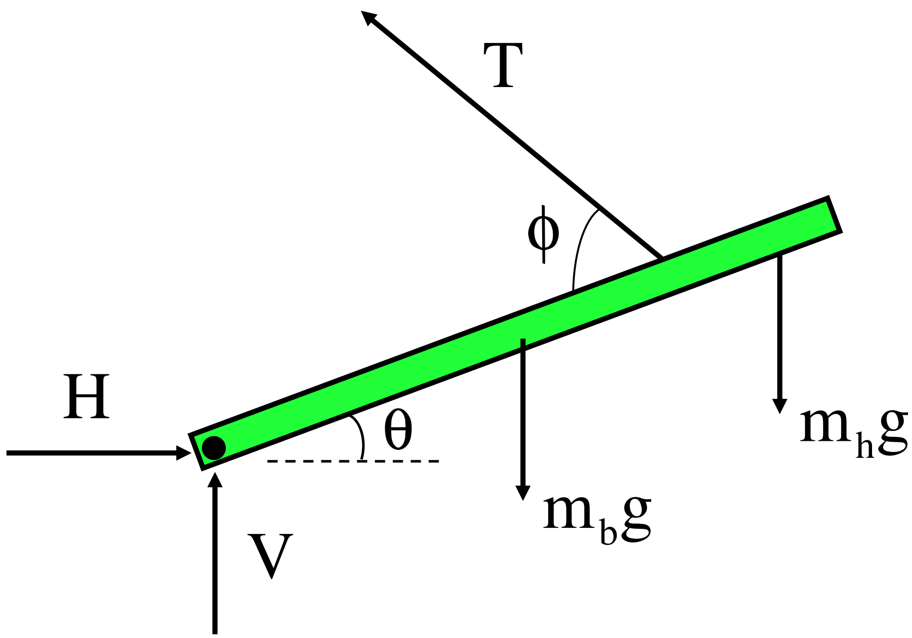
i.) What is the beam's *angular momentum* about the pin about that point?

2.) A beam of length "L" is pinned at an angle θ to a wall (see sketch). Tension in a rope "3L/4" from the pin keeps it in equilibrium. There is a massive lump a distance "5L/6" units up the beam. What is known is:

$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$



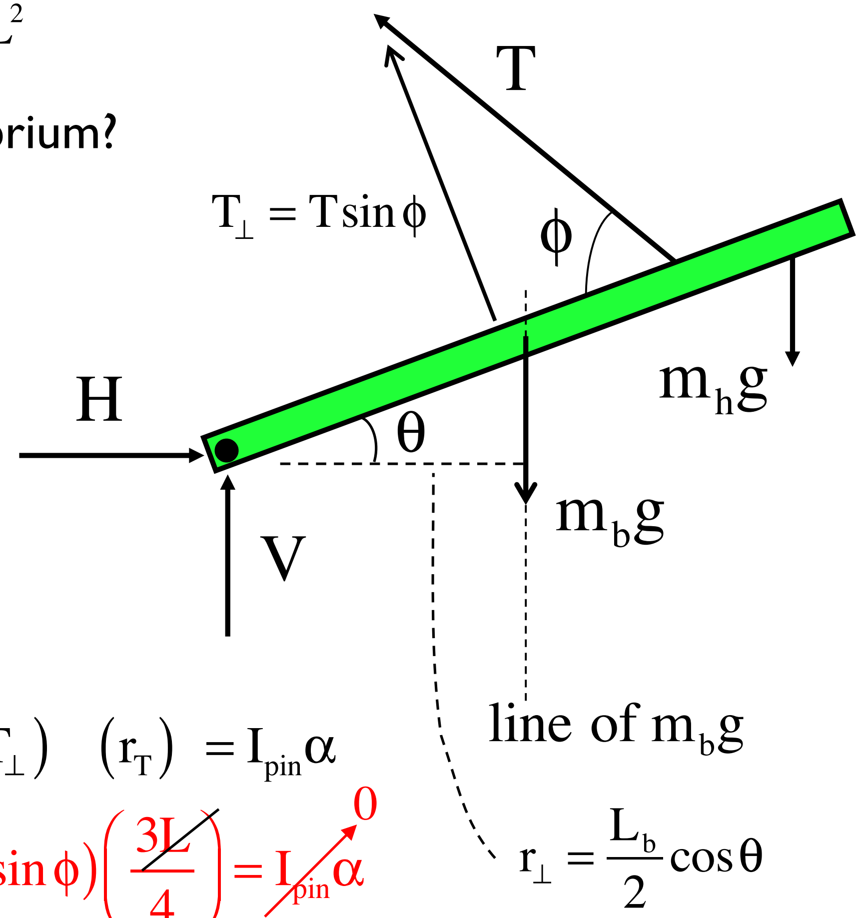
a.) Draw a f.b.d. for the forces on the beam.



$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

b.) What must the tension in the rope be for equilibrium?

As usual, this is a "torque about the pin" problem with the *angular acceleration* equal to zero.



$$\sum \Gamma_{pin}:$$

$$\cancelto{0}{\Gamma_H} + \cancelto{0}{\Gamma_V} - m_b g \left(r_{\perp,b}\right) - m_b g \left(r_{\perp,h}\right) + \left(T_\perp\right)\left(r_T\right) = I_{pin}\alpha$$

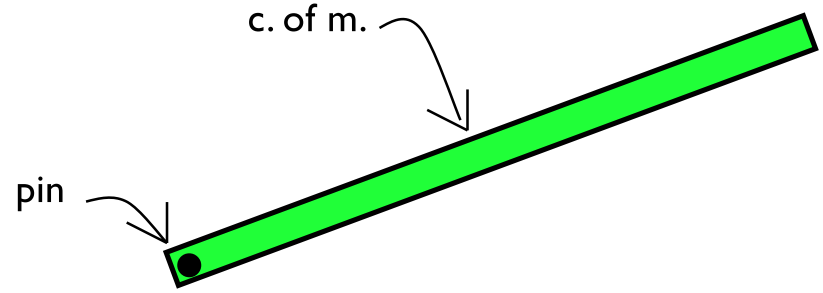
$$-m_b g\left(\frac{\cancel{L}}{2}\cos\theta\right) - m_h g\left(\frac{5\cancel{L}}{6}\cos\theta\right) + \left(T\sin\phi\right)\left(\frac{3\cancel{L}}{4}\right) = \cancelto{0}{I_{pin}\alpha}$$

$$\Rightarrow \quad T = \frac{\left(\frac{1}{2}\right) m_b g(\cos\theta) + \left(\frac{5}{6}\right) m_h g(\cos\theta)}{\left(\frac{3}{4}\right)\sin\phi}$$

$$\Rightarrow \quad T = \frac{6 m_b g(\cos\theta) + 10 m_h g(\cos\theta)}{9\sin\phi}$$

$$m_{beam}, L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_{beam} L^2$$

c.) Use the Parallel Axis Theorem to determine the *moment of inertia* <u>of the beam</u> about the pin.



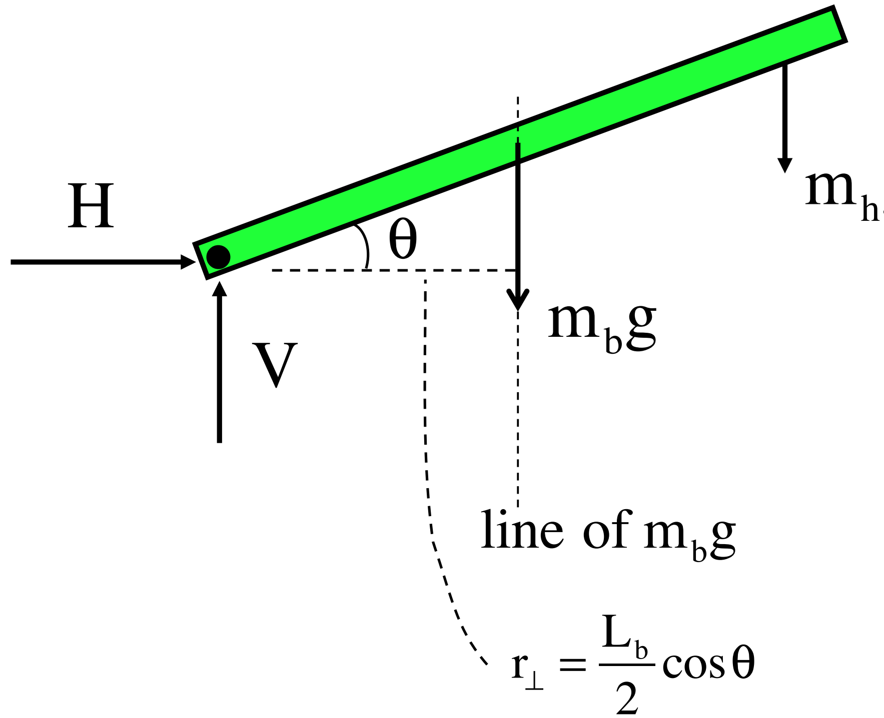
$$\begin{aligned} I_{p,beam} &= I_{cm} + m_b d^2 \\ &= \frac{1}{12} m_b L^2 + m_b \left(\frac{L}{2}\right)^2 \\ &= \frac{1}{3} m_b L^2 \end{aligned}$$

d.) Determine the *moment of inertia* <u>of the lump</u>, then system, about the pin.

$$\begin{aligned} I_h &= mr^2 \\ &= m_h \left(\frac{5}{6} L\right)^2 \\ &= \frac{25}{36} m_h L^2 \end{aligned}$$

$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

e.) The rope is cut and the beam begins to angular accelerate downward. Derive an expression for the beam's initial angular acceleration?



This is a "pure rotation" N.S.L. problem. In Part b, we summed up the torque about the pin. The only difference here is the term $I_{pin}\alpha$ isn't zero. Noting the distance "r" between the fixed pin and the *center of mass* is "L/2," and that $\alpha = \frac{a}{r} = \frac{a}{\left(L/2\right)}$, we can write:

$$\sum \Gamma_{pin}:$$

$$-m_b g\left(\frac{L}{2}\cos\theta\right) - m_h g\left(\frac{5L}{6}\cos\theta\right) = I_{total,pin}\alpha$$

$$-m_b g\left(\frac{L}{2}\cos\theta\right) - m_h g\left(\frac{5L}{6}\cos\theta\right) = \left(I_b + I_h\right)\alpha$$

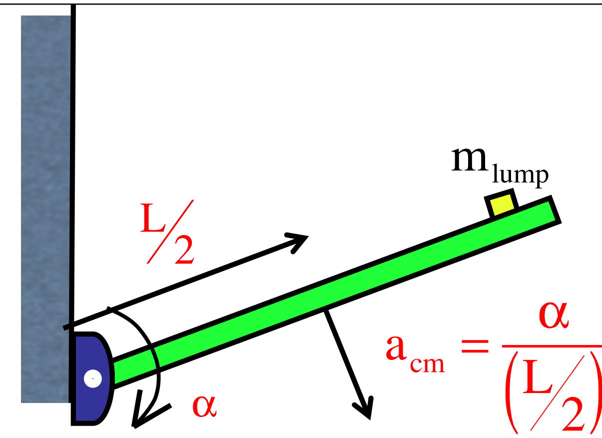
$$-m_b g\left(\frac{L}{4}\cos\theta\right) - m_h g\left(\frac{5L}{6}\cos\theta\right) = \left(\frac{1}{3} m_b L^2 + \frac{25}{36} m_h L^2\right)\left(\frac{a}{L/2}\right)$$

Get this far and I'm good . . .

$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

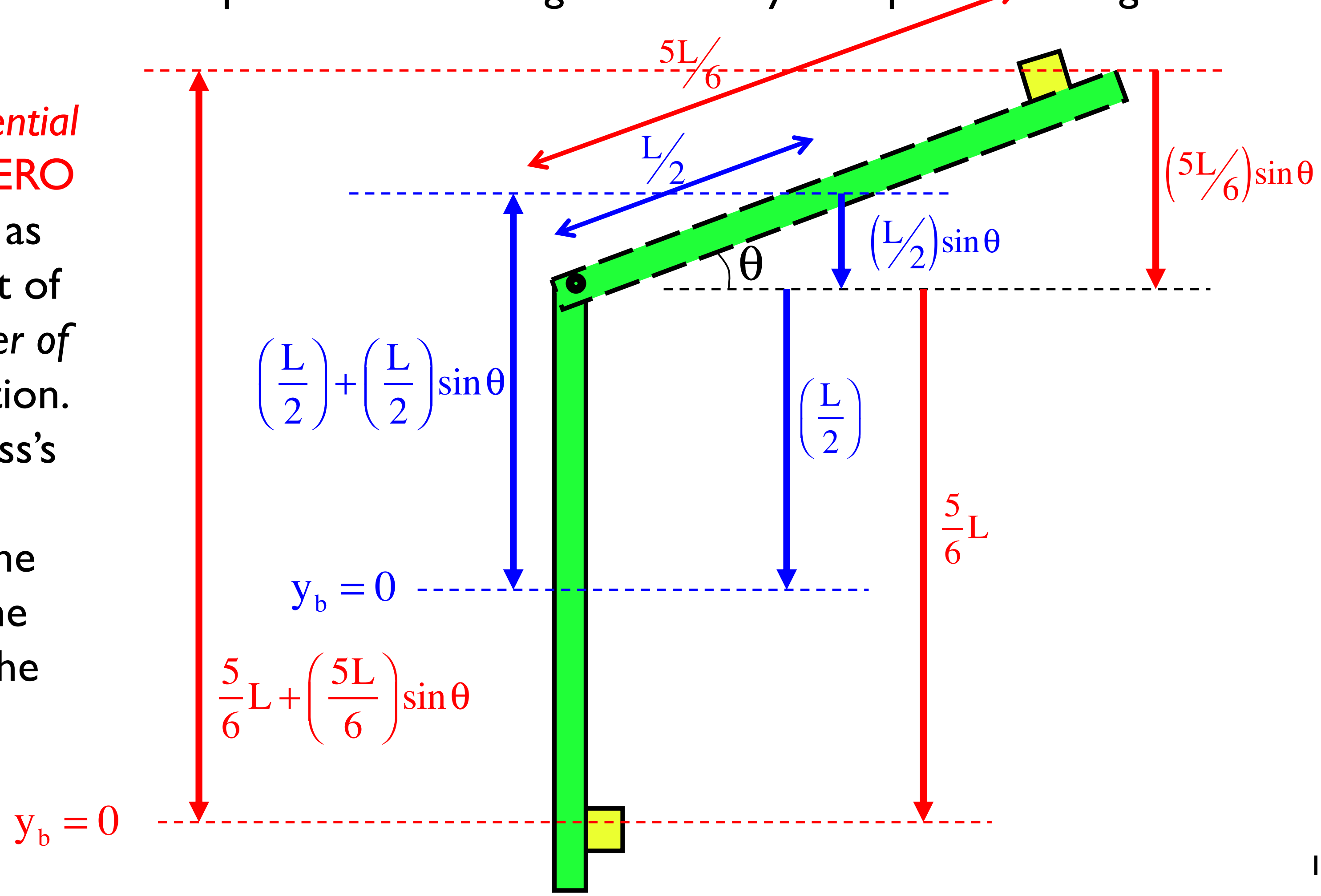
f.) What is the lump's initial *acceleration*?

$$a_{cm} = r_{cm}\alpha$$
$$= \left(\frac{L}{2}\right)\alpha$$



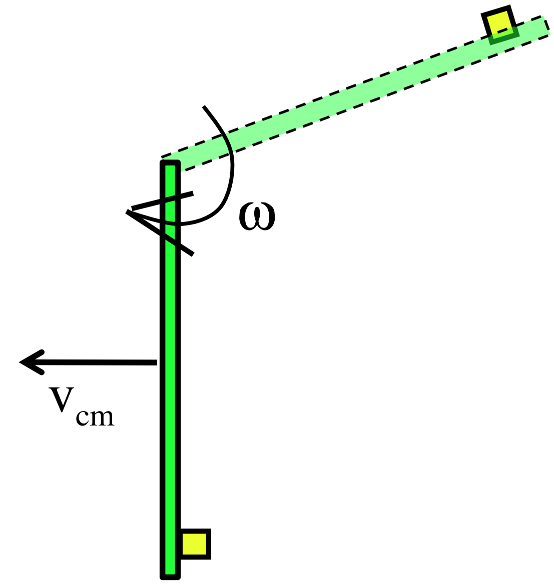
g.) The beam rotates about the pin. What is its angular velocity as it passes through the vertical?

The beam's *potential energy equals ZERO point* is defined as the lowest point of the beam's *center of mass* in the motion. The hanging mass's zero point is *its* lowest point. The sketch shows the zero point for the beam.



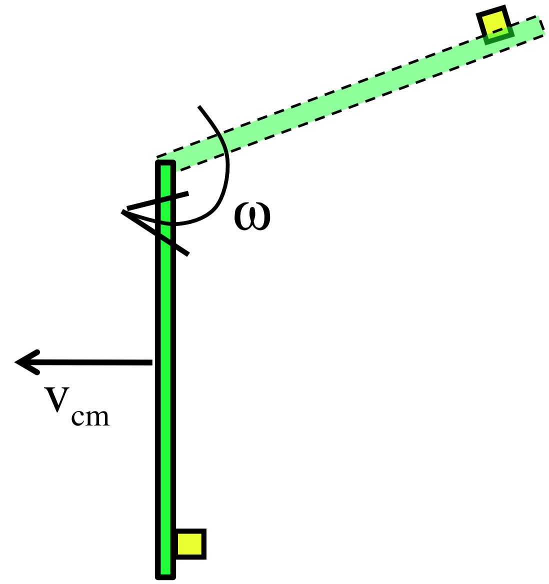
$$m_{beam}\,,\; m_{lump}\,,\; L, g, \theta, \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

g.) The beam rotates about the pin. What is its angular velocity as it passes through the vertical?



Get to the red relationship and I'll be happy!

m_{beam}, m_{lump}, L, g, θ, ϕ and $I_{cm,beam} = \frac{1}{12} m_b L^2$



g.) The beam rotates about the pin. What is its angular velocity as it passes through the vertical?

$$\sum KE_1 + \sum U_1 + \sum W_{ext} = \sum KE_2 + \sum U_2$$

$$0 + \left[m_b g\left(\frac{L}{2}+\frac{L}{2}\sin\theta\right) + m_h g\left(\frac{5}{6}L+\frac{5L}{6}\sin\theta\right)\right] + 0 = \left(\frac{1}{2}I_{pin}\omega^2 + \frac{1}{2}m_h\left(v_h\right)^2\right) + 0$$

$$\left[m_b g\left(\frac{L}{2}\right)(1+\sin\theta) + m_h g\left(\frac{5}{6}L\right)(1+\sin\theta)\right] = \left(\frac{1}{2}\left(\frac{1}{3}m_b L^2\right)\omega^2 + \frac{1}{2}m_h\left(\frac{5}{6}L^2\omega^2\right)\right)$$

$$\Rightarrow \qquad \omega = \sqrt{\left(\frac{6m_b + 10m_h}{2m_b + 5m_h}\right)\left(\frac{g}{L}\right)(1+\sin\theta)}$$

Get to the red relationship and I'll be happy!

$$m_{beam},\ m_{lump},\ L,\ g,\ \theta,\ \phi \text{ and } I_{cm,beam} = \frac{1}{12} m_b L^2$$

h.) What is the beam's center-of-mass velocity as it passes through the bottom of the arc?

$$v_{cm} = r_{cm}\omega$$
$$= \left(\frac{L}{2}\right)\omega$$

i.) What is the beam's angular momentum about the pin about that point?

$$L = I_{pin}\omega$$
$$= \left(\frac{1}{3} m_b L^2\right)\omega$$